

# Weak Charm Decays with Lattice QCD

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In this paper I review the status of lattice QCD calculations of  $D$  and  $D_s$  meson decay constants and of  $D$  meson semileptonic decay form factors. I restrict my discussion to results obtained from simulations with  $n_f = 2 + 1$  sea quarks.

## 1. Introduction and Motivation

Lattice QCD is the only systematically improvable calculational tool we have for quantitatively understanding nonperturbative QCD effects. Accurate theoretical calculations of nonperturbative QCD effects are essential for the experimental flavor physics program. One set of goals of the experimental program are accurate determinations of the CKM matrix elements. This is illustrated for the weak decay process  $D \rightarrow K l \nu$ . The experimentally measured (differential) decay rate can be written as

$$\frac{d\Gamma}{dq^2} = (\text{known}) |V_{cs}|^2 f_+^2(q^2) \quad (1)$$

where  $f_+(q^2)$  is one of the hadronic form factors which parameterize the hadronic matrix element for this process,  $\langle K | V_\mu | D \rangle$ . Hence, to determine  $|V_{cs}|$  from experimental measurements, we need a theoretical calculation of the form factor with matching precision. Another set of goals is to constrain beyond the standard model theories and to search for new physics signals. This effort complements the experiments at the high energy frontier. Accurate theoretical calculations are again essential.

Since lattice QCD calculations are complicated and time consuming, a third important goal specifically of the charm physics experiments is to test lattice QCD methods. For example, we can use Eq. 1 to determine the form factors from experimental measurements after taking  $|V_{cs}|$  from other sources. These tests are important to establish lattice methods for the  $B$  meson system, where the CKM matrix elements are less well known, and where input from lattice QCD is essential. The leptonic and semileptonic  $D$  meson decays discussed in this talk are ideal for this. They are not expected to be sensitive to new physics, and the corresponding hadronic matrix elements are straightforward to calculate. Once established, lattice QCD together with the experimental measurements can then be used to improve the determinations of the CKM angles  $V_{cd}$  and  $V_{cs}$ . All of these goals require accurate measurements and calculations.

## 1.1. Introduction to Lattice QCD

In lattice field theory, the space-time continuum is replaced by a discrete lattice. (For reviews of lattice QCD see Ref. [1].) This implies that derivatives are replaced by discrete differences, which in turn introduces discretisation errors into physical quantities. These errors generally vanish with a positive power of the lattice spacing ( $a$ ).

Nonperturbative calculations in lattice QCD can be performed using Monte Carlo methods. Lattice artifacts can be removed by reducing the lattice spacing used in numerical calculations. However, the computational cost increases as  $1/a^7$  (keeping the other parameters fixed). Alternatively, one can reduce discretisation errors by adding higher-dimensional operators to the action. This is called improvement. With improved actions the computational effort needed to perform reliable lattice QCD calculations can potentially be significantly reduced. This idea is behind much of the important progress made in lattice QCD calculations in recent years and has been an increasing part of research in lattice field theory.

The main obstacle for obtaining quantitative results (at the few percent level) from numerical simulations of lattice QCD has always been the computational effort associated with the proper inclusion of sea quark effects. Several years ago, substantial progress was made on this problem, in large part due to the development of an improved staggered fermion action [2]. For the first time, computationally efficient lattice simulations with realistic sea quark effects have become possible.

## 1.2. Light Quark Methods

The simplest lattice quark action replaces the covariant derivative in the continuum action by a discrete difference operator. This so-called naive action suffers from the doubling problem. For every continuum quark flavor, it has 15 additional unphysical flavors, called tastes. The staggered quark action combines four of these tastes into one Dirac field, by staggering the quark fields on a hypercube. This leaves four unphysical flavors (tastes). This action suffers from large  $O(a^2)$  lattice spacing artifacts due to taste

changing interactions. The Asqtad action is an improved staggered action where all tree-level discretisation errors are removed [2]. Its leading lattice spacing errors are therefore of  $O(\alpha_s a^2)$  and greatly reduced compared to the original staggered action. The Asqtad action is the computationally most efficient light quark action available. However, in order to use it for the sea quarks in numerical simulations, the unphysical flavors must be removed. The sea quarks are present in the fermion determinant of the path integral. To simulate two degenerate light (up and down) sea quarks, the MILC collaboration simply takes the square root of the light quark fermion determinant. For the strange sea quark, they take the fourth root of the determinant. This procedure is still somewhat controversial, but there is a growing body of evidence that its effects are controllable and disappear in the continuum limit [3]. The Asqtad action with the square root trick has been extensively tested in numerical simulations, most prominently in Ref. [4].

The HISQ (highly improved staggered quark) action is another version of an improved staggered action [5]. Like the Asqtad action, it removes all tree-level  $O(a^2)$  errors. The  $O(\alpha_s a^2)$  errors in the Asqtad action are rather large, due to taste changing interactions which appear at one-loop order. The HISQ action reduces the  $O(\alpha_s a^2)$  taste-changing effects by roughly a factor of three over the Asqtad action. The HISQ action has not yet been used to generate  $n_f = 2 + 1$  sea quark ensembles. Its computational cost is expected to be about a factor of two larger than the Asqtad action.

Other light quark methods include the Wilson action and its improvements [6], Domain Wall Fermions [7] and Overlap fermions [8], with increasing computational cost. The Wilson action solves the doubling problem by adding a dimension five operator which breaks chiral symmetry. Domain Wall Fermions solve the doubling problem by adding a fifth dimension, while keeping chiral symmetry almost exact. Overlap fermions have exact chiral symmetry, but a complicated operator structure.

### 1.3. Heavy Quark Methods

On the lattice, heavy quarks with  $am_Q$  large, are best treated within an effective field theory framework (NRQCD or HQET). One can start with an effective field theory, and discretise it as in Ref. [11], for example. Alternatively, one can start with a relativistic lattice action and analyze its mass dependent discretisation errors using effective field theories. The charm quark is too light for a straightforward implementation of the former approach, so we will focus on the latter.

The Fermilab approach [12] starts with the improved relativistic Wilson action [6] and the observa-

tion that the Wilson action has the same heavy quark limit as QCD. With a simple prescription, the Wilson action can be used for heavy quarks without errors that grow with the heavy quark mass,  $(am_Q)^n$ . This approach can be used for both charm and beauty quarks. With the improved Wilson action, the leading discretisation errors are  $O(\alpha_s \Lambda/m_Q)$  and  $O(\Lambda/m_Q)^2$ .

The HISQ action is so highly improved that it can be used for charm quarks with an additional tuning of a parameter in the action, provided that the lattice spacing is small enough [5]. The leading mass dependent discretisation errors are formally of order  $O(\alpha_s(am_c)^2)$  and  $O(am_c)^4$ .

### 1.4. Systematic Errors

The most important sources of systematic error in lattice QCD calculations are sea quark effects; using unphysically large masses for the up and down quarks; discretisation effects; finite volume effects; and renormalisation effects.

In order to be phenomenologically relevant, a lattice QCD calculation must use gauge configurations that include the effects of three light sea quarks. Since the masses of the up and down quarks are generally taken to be degenerate, this is also referred to as the  $n_f = 2 + 1$  case.

Until roughly five years ago, almost all lattice QCD calculations used ensembles generated either in the quenched approximation or with an incorrect number of sea quarks (generally,  $n_f = 2$ ) because of the computational cost associated with including sea quarks in the simulations. The quenched approximation omits sea quark effects entirely, at the cost of adding a systematic error in the range of 10 – 30% for physical quantities involving stable hadrons [4]. This error must be determined on a case by case basis. Simulations with an incorrect number of sea quarks carry a similar systematic error, which is hard to estimate *a priori*.

The computational cost increases with decreasing sea quark mass as  $m_l^{-2.5}$ . All simulations to date use masses for the light sea quarks which are larger than the physical up and down quark masses. (Note, the strange quark mass is large enough to be simulated at its physical value.) We can use chiral perturbation theory (ChPT) to guide the extrapolations from the light sea quark masses used in the simulations to the physical masses. ChPT is an effective theory of QCD, which can be applied to (lattice QCD calculations involving) pions and kaons. It can be combined with heavy quark effective theory and be applied to heavy-light systems, such as  $D$  and  $B$  mesons. Furthermore, it can be extended to include the leading light quark discretisation errors. Indeed, this has been done for the taste changing interactions of the Asqtad action and is called staggered ChPT (SChPT) [10].

The ChPT extrapolations are a significant but controllable source of systematic error. In order to keep this error at the few percent level or less, one needs to include simulations with a range of light sea masses, keeping  $m_l < m_s/2$ . The lattice QCD calculations described here include light sea quarks with masses in the range  $m_l = 1/10 m_s - 1/2 m_s$ . Hence, the lightest light sea quark masses are only a factor of 2–3 away from their physical value.

As described in the previous sections, the lattice actions give rise to discretisation errors. They can usually be estimated *a priori* using power counting arguments. However, even with improved actions, it is important to study and possibly remove these errors by repeating the calculation at several lattice spacings.

## 1.5. Simulation Parameters

The simulation parameters of the  $n_f = 2 + 1$  sea quark ensembles generated by the MILC collaboration using the Asqtad action (with the square root trick) are listed below and are shown in Figure 1. Each ensemble contains between 450–800 configurations. The ensembles contain one sea quark with a mass near the strange quark mass,  $m_s$ , and two degenerate light sea quarks with masses,  $m_l$ .

- $a = 0.15$  fm;  $m_l = 0.1 m_s, 0.2 m_s, 0.4 m_s, 0.6 m_s$ .
- $a = 0.12$  fm;  $m_l = 0.125 m_s, 0.25 m_s, 0.5 m_s, 0.75 m_s$ .
- $a = 0.09$  fm;  $m_l = 0.1 m_s, 0.2 m_s, 0.4 m_s$ .

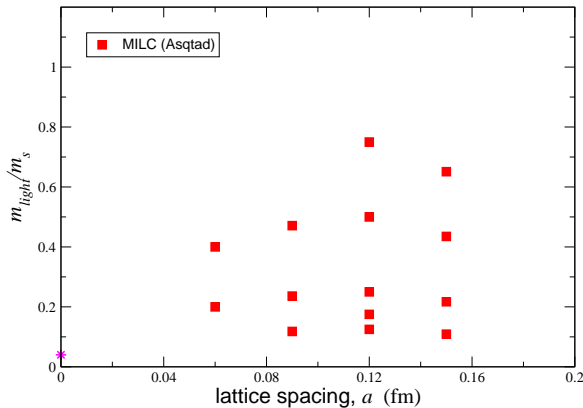


Figure 1: Simulation parameters for the MILC ensembles with  $n_f = 2 + 1$ , showing  $m_l/m_s$  vs. lattice spacing  $a$  (red squares). The physical point is at  $m_l/m_s = 1/25$  (pink burst).

## 2. Semileptonic $D$ Meson Form Factors

The semileptonic decays  $D \rightarrow K(\pi)l\nu$  are mediated by weak vector currents. The hadronic matrix elements for semileptonic decays are parameterized in terms of form factors. In our case there are two form factors, conventionally  $f_+(q^2)$  and  $f_0(q^2)$ . The form factors are functions of the virtual  $W$  boson momentum transfer,  $q^2$ , or, equivalently, the recoil momentum of the daughter meson. This introduces additional lattice spacing errors:

$$\langle K|V_\mu|D \rangle^{\text{lat}} = \langle K|V_\mu|D \rangle^{\text{cont}} + O(ap_K)^n \quad (2)$$

Hence, discretisation errors are smallest, when  $p_K$  is small and  $q^2 \approx q_{\text{max}}^2 = (m_D - m_K)^2$ .

The finite lattice volume provides an infrared cut-off, and therefore a minimum value for finite momentum,  $p_{\text{min}} = \frac{2\pi}{L}$ . Lattice three-momenta can be written in terms of  $p_{\text{min}}$  as  $\vec{p} = p_{\text{min}}(n_x, n_y, n_z)$ , where  $n_x, n_y, n_z$  are integers. For example, for  $a = 0.1$  fm,  $L = 20$ ,  $p_{\text{min}} = 620$  MeV.

To date, the only lattice results for semileptonic  $D$  meson form factors with  $n_f = 2 + 1$  are from the Fermilab Lattice and MILC collaborations [13]. They use the MILC  $a = 0.12$  fm lattices with light sea quark masses in the range  $m_l = 1/8 m_s - 3/4 m_s$ , the Asqtad action for the light valence quarks and the Fermilab action for the charm quark. Staggered chiral perturbation theory is used to extrapolate to the physical light quark masses and to remove the leading discretisation errors due to taste violations.

Figure 2 shows a comparison of the lattice QCD result for the normalization  $f_+^K(0)$  for  $D \rightarrow Kl\nu$  with experimental determinations (where  $V_{cs}$  is taken from other sources). The results are in very good agreement; however, the lattice QCD result has much larger errors than the experimental determinations. The comparison between lattice theory and experiment for  $f_+^\pi(0)$  is similar [14].

The shape of the form factor can also be determined in lattice QCD. However, in Ref. [13] the form factors were calculated at a few values of recoil momentum. Then the BK [16] parameterisation was used to determine the  $q^2$  dependence of the form factors. Since the errors increase with recoil momentum, the shape of the form factors is fixed mainly from the form factors near  $q_{\text{max}}^2$  and from using the BK parameterisation [15]. The lattice QCD result appeared before the new measurements by the FOCUS [17] and Belle [18] collaborations were announced, so it is one of a very few lattice QCD *predictions*. Figure 3 [15] shows a comparison of the lattice prediction for the  $q^2$  dependence with experimental data from the Belle collaboration [18]. The agreement is excellent. However, a quantitative comparison between the BK shape parameter determined from experiment and lattice theory is difficult to interpret, as eloquently argued by Richard

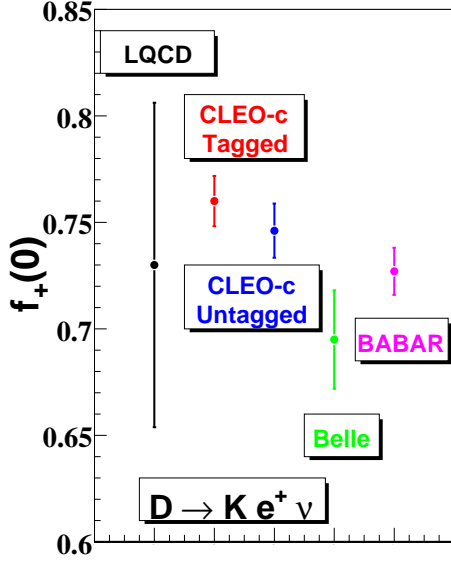


Figure 2:  $f_+^K(0)$  in comparison from Ref. [14]

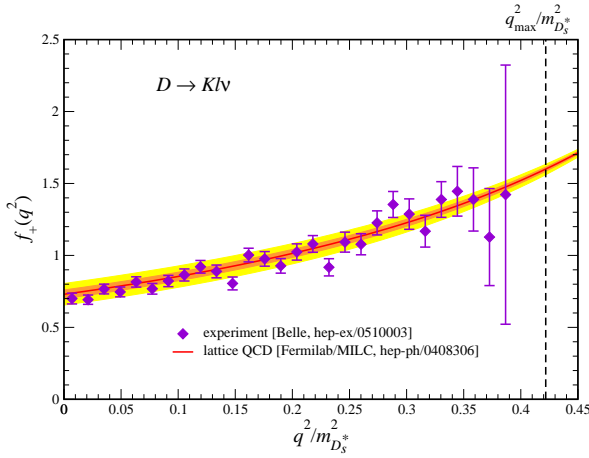


Figure 3: The shape of  $f_+^K(q^2)$  in comparison. The lattice prediction for the shape is from Ref. [15].

Hill [19]. A model independent parameterisation of the shape based on the  $z$ -expansion would avoid this difficulty [20]. The  $z$ -expansion is being used by the Fermilab Lattice and MILC (FNAL/MILC) collaboration to parameterize the  $q^2$  dependence of the form factors for  $B \rightarrow \pi l \nu$  [21]. This works quite well, as shown in Figure 4. Any new lattice analysis of semileptonic  $D$  meson decay form factors will (should) adopt the  $z$ -expansion to determine the shape.

A number of additional improvements are possible in future calculations. Twisted boundary conditions

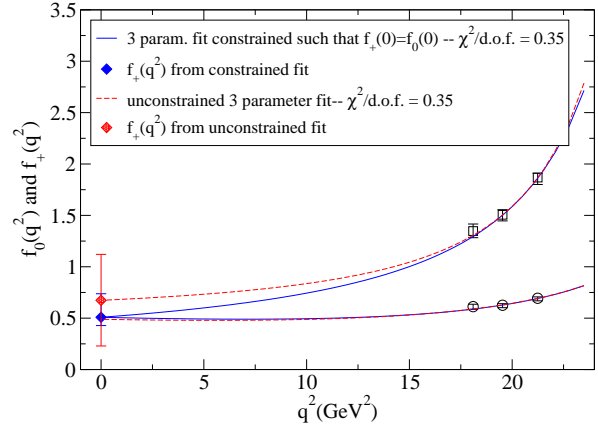


Figure 4: The form factors for  $B \rightarrow \pi l \nu$  vs.  $q^2$  from fitting to the  $z$ -expansion. The preliminary results are from Ref. [21].

can be used to adjust the lattice momenta to arbitrarily small values [22], which would improve the shape determination. Double ratio methods similar to what has been developed for lattice studies of  $B \rightarrow D l \nu$  [23] can be adapted to semileptonic  $D$  meson decays [24]. This may lead to reduced statistical errors as well as improvements of some of the systematic errors.

### 3. Leptonic Decay Constants $f_D$ and $f_{D_s}$

Charm leptonic decays provide another important test of lattice QCD. The lattice methods for calculating decay constants in the charm and beauty meson systems are the same. Indeed, with the Fermilab approach one uses the same heavy quark action in both systems and the heavy quark discretisation errors are expected to be larger for  $D$  mesons than for  $B$  mesons.

There are now results from two groups (FNAL/MILC and HPQCD). Both use the MILC ensembles at  $a = 0.09$  fm, 0.12 fm, 0.15 fm.

The first FNAL/MILC results came out in 2005 [25], just days before CLEO-c announced its first precise determination of  $f_{D^+}$  [26]; the two results were in good agreement.

The HPQCD collaboration announced their results for decay constants with much reduced errors this summer [27] and FNAL/MILC presented updated results at the Lattice 2007 conference [28], also with reduced errors. The new FNAL/MILC analysis was done “blind”, where an overall unknown offset was added to the lattice data. The final results were unblinded shortly before they were presented at the Lattice 2007 conference, making this the first (intentionally) blind lattice analysis. Table I compares the main

features of the two calculations. More details about the HPQCD and FNAL/MILC calculations, including discussion of the error analysis and plots of chiral and lattice spacing extrapolations can be found in Refs. [29] and [28] respectively. The FNAL/MILC analysis includes more lattice ensembles, more valence quark masses per ensemble, and uses staggered chiral perturbation theory (Staggered ChPT) to remove the leading light quark discretisation errors. The HPQCD collaboration considers only the case  $m_q = m_l$ , where  $m_q$  denotes the light valence quark mass and  $m_l$  denotes the light sea quark mass. They use continuum ChPT with generic  $O(a^2)$  terms added in the chiral fits.

The main difference between the two calculations is the valence quark actions. The HPQCD collaboration uses the HISQ action for all (charm, strange and light) valence quarks, whereas the FNAL/MILC collaboration uses the Fermilab action for the charm quarks and the Asqtad action for the strange and light valence quarks. Since the HISQ action is more improved than the Fermilab action, the HPQCD result has much smaller heavy quark discretisation errors. This is the main reason for the difference in the total errors between the two results.

Table II compares the error budgets for the 2005 FNAL/MILC calculation with the FNAL/MILC Lattice 2007 one. The error reduction is mainly due to including three MILC ensembles at  $a = 0.09$  fm (and 8-12 different valence masses). This reduces the heavy quark and light quark discretisation errors, and better constrains the staggered ChPT.

Figures 5, 6, and 7 compare the lattice results for  $f_{D^+}$ ,  $f_{D_s}$ , and  $f_{D_s}/f_{D^+}$ , respectively, to the corresponding experimental averages. The experimental averages are from Ref. [14]. The new CLEO-c result  $f_{D_s} = 275 \pm 10 \pm 5$  presented at this conference by Steven Blusk [30], is very similar to Ref. [14].

The FNAL/MILC results agree with the experimental averages at the one sigma level. The HPQCD results agree very well with the FNAL/MILC results. There is a hint of disagreement between the HPQCD result for  $f_{D_s}$  and the experimental average at the two sigma level. However, the experimental determinations of the decay constants must assume a value for the CKM angle  $V_{cs}$  from other sources. We are approaching a level of precision, where tests of lattice QCD should be performed on CKM free quantities such as the ratio of semileptonic to leptonic decay rates suggested in Ref. [31].

## 4. Conclusions and Outlook

With the generation of the MILC ensembles, the stakes for lattice QCD calculations have risen. We are now able to calculate the simplest quantities to a few percent accuracy. As always, repetition is desirable

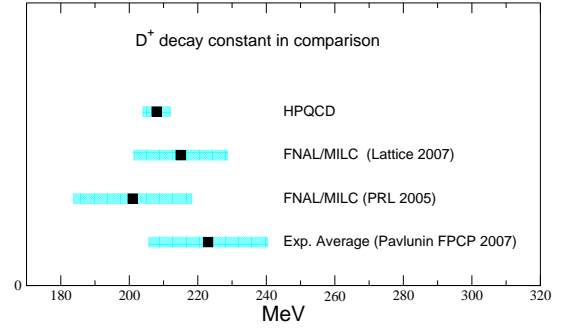


Figure 5: Comparison of lattice QCD results for  $f_{D^+}$  with experiment.

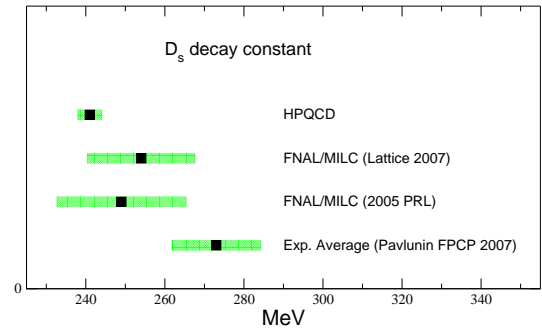


Figure 6: Comparison of lattice QCD results for  $f_{D_s}$  with experiment.

to test different lattice methods against each other. To date, all lattice calculations that include realistic sea quark effects use the MILC ensembles with rooted Asqtad sea quarks. As mentioned in section 1.2, the Asqtad action carries the smallest computational cost of any light quark action. Nevertheless, recently other collaborations have started to generate ensembles with different sea quark actions. An overview is given in Figure 8. It shows that the other ensembles are being generated at similar values of lattice spacing and light quark masses as the MILC ensembles. The MILC collaboration continues to generate new ensembles at even smaller lattice spacings. They are also generating additional configurations for the existing ensembles to further reduce statistical errors. As in experiment, in lattice QCD smaller statistical errors give better con-

Table I Comparison of the main features of the HPQCD and Fermilab Lattice/MILC calculations.

FNAL/MILC		HPQCD	
Fermilab action for charm quark		HISQ action for charm and light valence quarks	
Asqtad action for strange and light valence			
$a$ (fm)	$m_l/m_s$ sea quark	$a$ (fm)	$m_l/m_s$ sea quark
0.09	1/10, 1/5, 2/5	0.09	1/5, 2/5
0.12	1/8, 1/4, 1/2, 3/4	0.12	1/8, 1/4, 1/2
0.15	1/10, 1/5, 2/5, 3/5	0.15	1/5, 2/5
8 – 12 light valence quark masses per ensemble		1 valence quark mass/ensemble, $m_{\text{valence}} = m_{\text{sea}}$	
Partial nonperturbative renormalisation		Nonperturbative renormalisation from PCAC	
Staggered ChPT fits to all valence and sea quark ensembles together		Continuum ChPT + $O(a^2)$ terms fit to all ensembles together	
Blind analysis for Lattice 2007			

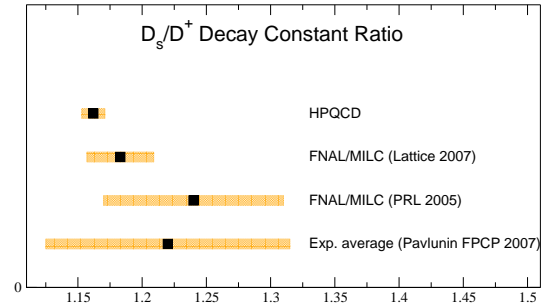
Table II Comparison of the error budget of the 2005 FNAL/MILC results with the Lattice 2007 results. Numbers are given in percent.

	PRL 2005 [25]		Lattice 2007 [28]	
source	$f_{D^+}$	$f_{D_s}/f_{D^+}$	$f_{D^+}$	$f_{D_s}/f_{D^+}$
statistics	1.5	0.5	3.8	1.0
HQ discretisation	4.2	0.5	2.7	0.3
light quark + Chiral fits	6.3	5	2.7	1.8
inputs ( $a$ , $m_c$ , $m_s$ )	2.8	0.6	3.4	0.5
higher order PT	1.3	1.3	0.3	-
+ other small sources (finite volume, ...)				
total systematic	8.5	5.4	5.3	2.0

trol over systematic errors. Hence, lattice calculations based on the MILC ensembles will continue to become more accurate.

Any modern lattice QCD calculation that claims phenomenological relevance must include a serious systematic error analysis. To be relevant, it must include the correct number of sea quarks (which all the ensembles shown in Figure 8 do). While the masses of the light sea quarks are still unphysically large, it must also include a study of the light quark mass dependence with sufficiently small sea quark masses. Among other sources of error, discretisation effects must be estimated and tested by repeating the calculation at more than one lattice spacing.

In summary, we should expect to see lattice results from these new ensembles in the near future. They will provide important consistency tests of the lattice methods, and in particular of the square root trick used by the MILC collaboration to generate their  $n_f = 2 + 1$  ensembles.

Figure 7: Comparison of lattice QCD results for  $f_{D_s}/f_{D^+}$  with experiment.

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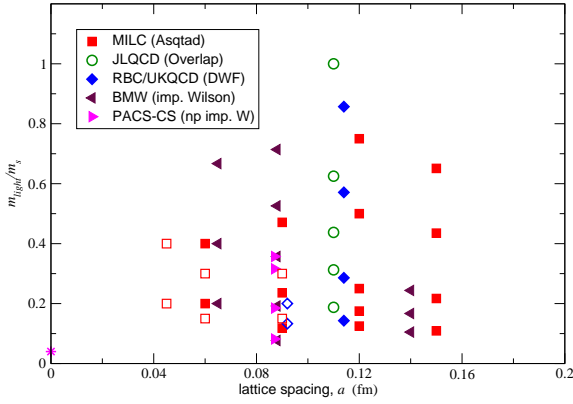


Figure 8: Simulation parameters for ensembles with  $n_f = 2 + 1$  showing  $m_l/m_s$  vs. lattice spacing  $a$ . Filled symbols denote existing ensembles. Unfilled symbols denote ensembles which are currently being generated or planned. Red squares: MILC [9], blue diamonds: RBC/UKQCD [32], purple left triangles: BMW (improved Wilson) [33], pink right triangles: PACS-CS (nonperturbatively improved Wilson) [34], green circles: JLQCD (Overlap) [35]. The physical point is at  $m_l/m_s = 1/25$  (pink burst).

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